# J-MAPS Technical Memorandum 08-12

# Star Counts and Photometric Parallaxes as a Function of Spectral Type and V-band Magnitude

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## ABSTRACT

Using B and V-band photometry from NOMAD, both raw counts and relative percentages of each spectral type per magnitude are calculated, as well as the average distance and parallax of each spectral type per magnitude to provide the Joint Milli Arcsecond Pathfinder Survey (J-MAPS) with an enhanced understanding of potential stars to observe.

## 1. Introduction

The goal of this study is to find the general distribution of stellar spectral types at each Johnson V-band apparent magnitude between -3 and 14, using the Naval Observatory Merged Astrometric Dataset (Zacharias et al. 2004; NOMAD) as the main database to realize expectations of target stars for the Joint Milli-Arcsecond Pathfinder Survey (J-MAPS). The B and V-band magnitudes of each star are used to calculate the spectral type, and thus, only stars with both B and V-band data were included in the results. Then, assuming all stars to be Zero Age Main Sequence Stars (ZAMS), each star's absolute V-band magnitude is calculated based on either $6^{th}$ order polynomial interpolations or spectral type averages, depending on the spectral type. With each star's absolute and apparent V-band magnitude, each star's photometric distance and parallax are calculated. The results include the percentage of stars with spectral data at each magnitude, the tally and percentage of stars of each spectral type at each magnitude, the average distance to stars of a spectral type at a certain magnitude, and the average photometric parallax of stars of a spectral type at a certain magnitude.

## 2. Methodology

While NOMAD can provide up to 22 separate pieces of information about each star in the catalog, we are only interested in spectral distribution at a relatively coarse level of accuracy. Only the B-band and the V-band photometric data are neccessary to calculate the spectral types for this level of accuracy. First, all stars with a V-band magnitude brighter than 17 are extracted from NOMAD into a separate file. While J-MAPS is only required to observe 90% of stars at the $14^{th}$ magnitude, it will observe stars at fainter magnitudes without specifically allocating resources to do so. Stars with magnitudes as faint as 17 are extracted to ensure the validity of observed trends.

The B and V magnitudes of each star in the extraction file are then scanned and used to calculate a B-V color. The star's spectral type is determined using limitations on the B-V color established in Chapter 15 of Cox (2000) and listed in Table 1. The counts of the spectral types at magnitudes -3 through 16 were recorded in Table 2, while Table 3 shows the percentage of stars of a spectral type per magnitude. Table 4 shows the percentage of stars at each magnitude with sufficient photometric data for classification. All magnitudes are binned rounding to the nearest whole magnitude.

For spectral types B, A, F, G, and K, the same B-V data is used in three separate $6^{th}$ order polynomial interpolations from Mermilliod (1981), Schmidt-Kaler et al. (1982), and Turner (1981). Each scheme calculated the absolute magnitude with the same general equation, Equation 1, but different sets of coefficients, listed in Table 5.

$$M_v = a(B-V)^6 + b(B-V)^5 + c(B-V)^4 + d(B-V)^3 + e(B-V)^2 + f(B-V) + g \quad (1)$$

The resulting estimates of the absolute magnitudes of the three schemes are then averaged to increase accuracy, and the mean of the three approximations is used as the star's absolute magnitude for types B through G.

However, while this study excluded data from NOMAD that had been flagged as insufficient or inaccurate, the photometric data available for O stars is not always sufficient to use the interpolations, a method that resulted in frequent disagreement with empirical data from Cox (2000) on real O stars. Thus, a spectral class of O9 is assigned because it is the most common among O stars. The corresponding ZAMS absolute V-band magnitude of -4.5, according to chapter 15 of Cox (2000), was then applied to each O star. This method is applied to the K and M stars for the same reason, using the common spectral types of K4 and M0 and the respective magnitudes of 7.0 and 8.8, according to Appendix E of Carroll (1996).

With values for both the absolute ($M_v$) and the apparent ($m_v$) V-band magnitudes, the photometric distance ($d$) in parsecs is found using the distance modulus relation:

$$d = 10^{\frac{(m_v - M_v + 5)}{5}} \tag{2}$$

This approach can be problematic when the star in question is not on the main sequence, causing the star's actual absolute magnitude to vary significantly from the prediction. This issue affects K and M stars the most because giants of the same spectral types have much brighter absolute magnitudes than the corresponding dwarves. Because the distance modulus is only accurate when the estimate of the absolute magnitude is accurate, additional strict infrared photometric constraints from Chapter 7 of Cox (2000), listed in Table 6, were used in an attempt to include in the distance calculations only K and M stars definitely on the main sequence. However, even with the additional constraints, some giants' photometric values could still fall within the boundaries of main sequence stars, introducing error to some distances. The counts of the K and M stars at each magnitude that are used in the distance calculations are recorded in Table 7. From the distance, the photometric parallax ($\pi$, seconds of arc) is calculated using the relation:

$$\pi'' = \frac{1}{d} \tag{3}$$

The distance and parallax of all stars of a given spectral type at a given magnitude are averaged and recorded in Tables 8 and 9, respectively.

## 3. Results

Information for over 113 million stars is evaluated. NOMAD provides sufficient information on more than 97% of those stars to allow classification.

Of particular importance are four different V-band magnitudes: V=5, 10, 12, and 14. The normal saturation level of the detector occurs at V=5 magnitude. The J-MAPS guide stars are $10^{th}$ magnitude stars. The acceptable and normal limits of accuracy occur at the $12^{th}$ and $14^{th}$ magnitudes, respectively. Figures 1-4 show the spectral composition of these four magnitudes. The spectral distribution at the $5^{th}$ magnitude is roughly evenly spread between the spectral types, except for the small contribution from the O and G stars. At the $10^{th}$, $12^{th}$, and $14^{th}$ magnitudes, the F, G, and K stars comprise the vast majority. The percentage of M stars drops after the $11^{th}$ magnitude.

Table 1: Spectral Type B-V Constraints

| | O | B | A | F | G | K | M |
|---|---|---|---|---|---|---|---|
| Min. B-V | -10.1 | -0.30 | -0.02 | 0.30 | 0.58 | 0.81 | 1.40 |
| Max. B-V | -0.30 | -0.02 | 0.30 | 0.58 | 0.81 | 1.40 | 10.0 |

Reference: Table 1 values come from Cox (2000).

Table 2: Spectral Distribution by Magnitude
Raw Stellar Counts

| Apparent Visual Magnitude | O | B | A | F | G | K | M | Magnitude Total | Cumulative Total |
|---|---|---|---|---|---|---|---|---|---|
| -3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 3 | 2 | 0 | 0 | 2 | 3 | 10 | 11 |
| 2 | 1 | 24 | 16 | 4 | 2 | 8 | 12 | 67 | 78 |
| 3 | 0 | 58 | 38 | 13 | 7 | 45 | 31 | 192 | 270 |
| 4 | 3 | 147 | 126 | 68 | 13 | 183 | 65 | 605 | 875 |
| 5 | 2 | 410 | 426 | 198 | 58 | 503 | 292 | 1889 | 2764 |
| 6 | 3 | 850 | 1457 | 766 | 195 | 1778 | 879 | 5928 | 8692 |
| 7 | 2 | 1484 | 4372 | 2966 | 684 | 5440 | 2656 | 17604 | 26296 |
| 8 | 8 | 1946 | 11782 | 10638 | 2643 | 16399 | 7362 | 50778 | 77074 |
| 9 | 27 | 2258 | 27387 | 35241 | 9552 | 45827 | 19922 | 140214 | 217288 |
| 10 | 97 | 2610 | 54116 | 102329 | 40447 | 125387 | 50382 | 375368 | 592656 |
| 11 | 865 | 6100 | 105885 | 263555 | 159596 | 282126 | 143233 | 961360 | 1554016 |
| 12 | 19017 | 70867 | 307903 | 472741 | 404552 | 614303 | 151198 | 2040581 | 3594597 |
| 13 | 130454 | 237326 | 1017351 | 1848637 | 1422663 | 1506556 | 378569 | 6541556 | 10136153 |
| 14 | 396285 | 497027 | 2168410 | 3984197 | 3170289 | 4099727 | 1345018 | 15660953 | 25797106 |
| 15 | 1158190 | 1085527 | 3990546 | 6833319 | 5733768 | 7634168 | 2537313 | 28972831 | 54769937 |
| 16 | 2661315 | 2567142 | 7789796 | 12083110 | 10676571 | 14934463 | 4753449 | 55465846 | 110235783 |
| Total | 4366269 | 4473779 | 15479614 | 25637783 | 21621040 | 29266915 | 9390383 | 110235783 | ··· |

Note: Table 2 lists the number of stars of each spectral type at apparent visual magnitudes -3 through 16. The Magnitude Total column shows the total number of stars at every magnitude, while the Cumulative Total column lists the number of all stars at that magnitude and all brighter magnitudes. The row Total counts the total number of stars of each spectral type at every magnitude combined. The K star and M star columns include stars that were not used in the distance and parallax calculations because the stars could not be confirmed as main sequence.

Table 3: Spectral Distribution by Magnitude
Percentage

| Apparent Visual Magnitude | O | B | A | F | G | K | M |
|---|---|---|---|---|---|---|---|
| -3 | ... | ... | ... | ... | ... | ... | ... |
| -3 | ... | ... | ... | ... | ... | ... | ... |
| -2 | ... | ... | ... | ... | ... | ... | ... |
| -1 | 0.00 | 0.00 | 0.00 | 100.00 | 0.00 | 0.00 | 0.00 |
| 0 | ... | ... | ... | ... | ... | ... | ... |
| 1 | 0.00 | 30.00 | 20.00 | 0.00 | 0.00 | 20.00 | 30.00 |
| 2 | 1.49 | 35.82 | 23.88 | 5.97 | 2.99 | 11.94 | 17.91 |
| 3 | 0.00 | 30.21 | 19.79 | 6.77 | 3.65 | 23.44 | 16.15 |
| 4 | 0.50 | 24.30 | 20.83 | 11.24 | 2.15 | 30.25 | 10.74 |
| 5 | 0.11 | 21.72 | 22.56 | 10.49 | 3.07 | 26.64 | 15.47 |
| 6 | 0.05 | 14.32 | 24.54 | 12.90 | 3.28 | 29.95 | 14.81 |
| 7 | 0.01 | 8.41 | 24.79 | 16.82 | 3.88 | 30.84 | 15.06 |
| 8 | 0.02 | 3.83 | 23.17 | 20.92 | 5.20 | 32.24 | 14.48 |
| 9 | 0.02 | 1.61 | 19.49 | 25.08 | 6.80 | 32.61 | 14.18 |
| 10 | 0.03 | 0.69 | 14.37 | 27.18 | 10.74 | 33.30 | 13.38 |
| 11 | 0.09 | 0.63 | 11.00 | 27.39 | 16.59 | 29.32 | 14.89 |
| 12 | 0.93 | 3.47 | 15.07 | 23.14 | 19.80 | 30.07 | 7.40 |
| 13 | 1.99 | 3.62 | 15.53 | 28.22 | 21.71 | 23.00 | 5.78 |
| 14 | 2.52 | 3.17 | 13.82 | 25.39 | 20.20 | 26.12 | 8.57 |
| 15 | 3.99 | 3.74 | 13.74 | 23.54 | 19.75 | 26.29 | 8.74 |
| 16 | 4.79 | 4.62 | 14.02 | 21.74 | 19.21 | 26.87 | 8.55 |

Note: Table 3 presents the percentage of stars at a given magnitude of each spectral type.

Table 4: Percentage of Stars with B
and V-band Spectral Data by Magnitude

| Apparent Visual Magnitude | Number With Spectral Data | Number Without Spectral Data | Percentage With Spectral Data |
|---|---|---|---|
| -3 | 0 | 0 | ... |
| -2 | 0 | 0 | ... |
| -1 | 1 | 0 | 100.00 |
| 0 | 0 | 0 | ... |
| 1 | 10 | 0 | 100.00 |
| 2 | 71 | 0 | 100.00 |
| 3 | 191 | 0 | 100.00 |
| 4 | 601 | 0 | 100.00 |
| 5 | 1889 | 1 | 99.95 |
| 6 | 5937 | 66 | 98.90 |
| 7 | 17639 | 182 | 98.98 |
| 8 | 50857 | 332 | 99.35 |
| 9 | 140536 | 1397 | 99.02 |
| 10 | 376484 | 4508 | 98.82 |
| 11 | 962178 | 10596 | 98.91 |
| 12 | 2042902 | 38759 | 98.14 |
| 13 | 6551628 | 132212 | 98.02 |
| 14 | 15694871 | 356202 | 97.78 |
| 15 | 29033322 | 764913 | 97.43 |
| 16 | 55578537 | 1613989 | 97.18 |

Note: Table 4 records the percentages of stars for which NOMAD has B and V-band data because the B-V index was used initially to classify the stars' spectral type. Without B or V-band data, a star could not be classified.

Table 5: Interpolation Coefficients

| Author | a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|---|
| Mermilliod 1981 | 70.886 | -108.12 | 14.865 | 41.256 | -17.994 | 6.504 | 1.5854 |
| Schmidt-Kaler et al.1982 | -6.472 | 32.247 | -56.115 | 41.648 | -13.35 | 7.7839 | 0.8925 |
| Turner 1981 | -26.249 | 77.943 | -90.138 | 48.264 | -10.321 | 5.762 | 1.4555 |

Note: Interpolation method not used with spectral types O, K, or M because of the inaccuracy of O, K, and M star photometric data.

Table 6: Photometric Constraints for Main Sequence K and M Stars

| Color Index | K (mag) | M (mag) |
|---|---|---|
| Min. V-R | 0.64 | ⋯ |
| Max. V-R | 0.99 | 1.8 |
| Min. V-K | 1.96 | 3.65 |
| Max. V-K | 3.16 | 7.4 |
| Min. J-H | 0.45 | 0.62 |
| Max. J-H | 0.66 | 0.66 |

Note: Table 6 lists the extra photometric constraints from Cox (2000) placed on K and M stars to include in the distance and parallax calculations only those that are definitely on the main sequence.

Table 7: Counts of Main Sequence K and M Stars used in Distance and Parallax Calculations

| Magnitude | K | M |
|---|---|---|
| -3 | 0 | 0 |
| -2 | 0 | 0 |
| -1 | 0 | 0 |
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 2 | 0 | 1 |
| 3 | 1 | 0 |
| 4 | 65 | 0 |
| 5 | 177 | 0 |
| 6 | 398 | 4 |
| 7 | 1751 | 5 |
| 8 | 8295 | 192 |
| 9 | 24352 | 198 |
| 10 | 59926 | 275 |
| 11 | 91339 | 613 |
| 12 | 82544 | 427 |
| 13 | 48522 | 337 |
| 14 | 67413 | 2050 |
| 15 | 154377 | 7394 |
| 16 | 383218 | 25282 |

Note: Table 7 lists the counts of main sequence K and M stars used in distance and parallax calculations because only main sequence stars could be used in those calculations. Not all the K and M stars in NOMAD could be confirmed to be main sequence.

Table 8: Average Distance by Magnitude
(Parsecs)

| Apparent Visual Magnitude | O | B | A | F | G | K | M |
|---|---|---|---|---|---|---|---|
| -3 | ··· | ··· | ··· | ··· | ··· | ··· | ··· |
| -2 | ··· | ··· | ··· | ··· | ··· | ··· | ··· |
| -1 | ··· | ··· | ··· | ··· | ··· | ··· | ··· |
| 0 | ··· | ··· | ··· | ··· | ··· | ··· | ··· |
| 1 | ··· | 17.63 | 7.23 | ··· | ··· | ··· | ··· |
| 2 | 48.48 | 23.90 | 12.09 | 4.59 | 2.47 | ··· | 0.52 |
| 3 | ··· | 40.14 | 15.87 | 7.87 | 3.63 | 1.75 | ··· |
| 4 | 119.93 | 62.78 | 27.94 | 13.05 | 5.83 | 2.69 | ··· |
| 5 | 243.38 | 90.53 | 43.64 | 20.53 | 9.51 | 4.12 | ··· |
| 6 | 309.69 | 130.92 | 69.43 | 32.37 | 15.93 | 6.53 | 2.89 |
| 7 | 503.77 | 189.74 | 106.75 | 51.13 | 25.05 | 10.94 | 4.74 |
| 8 | 823.64 | 279.38 | 164.34 | 79.35 | 40.17 | 16.65 | 7.59 |
| 9 | 1364.37 | 434.10 | 253.02 | 122.78 | 64.46 | 26.35 | 10.72 |
| 10 | 2138.13 | 725.60 | 381.53 | 188.33 | 101.76 | 41.36 | 18.25 |
| 11 | 3492.49 | 1511.97 | 583.43 | 296.21 | 154.52 | 64.64 | 28.53 |
| 12 | 5551.82 | 3371.07 | 984.01 | 476.47 | 243.80 | 99.32 | 41.19 |
| 13 | 8393.02 | 5481.83 | 1550.78 | 762.69 | 388.19 | 158.22 | 74.67 |
| 14 | 13237.90 | 8832.10 | 2389.87 | 1176.27 | 606.28 | 260.71 | 116.45 |
| 15 | 21060.86 | 14480.41 | 3898.07 | 1880.72 | 959.00 | 418.44 | 185.60 |
| 16 | 32776.65 | 23073.86 | 6195.73 | 2970.78 | 1517.80 | 654.04 | 291.30 |

Note: Table 8 presents the average distance in parsecs to different spectral types at different magnitudes.

Table 9: Average Photometric Parallax by Magnitude
(mas)

| Apparent Visual Magnitude | O | B | A | F | G | K | M |
|---|---|---|---|---|---|---|---|
| -3 | ··· | ··· | ··· | ··· | ··· | ··· | ··· |
| -2 | ··· | ··· | ··· | ··· | ··· | ··· | ··· |
| -1 | ··· | ··· | ··· | ··· | ··· | ··· | ··· |
| 0 | ··· | ··· | ··· | ··· | ··· | ··· | ··· |
| 1 | ··· | 56.72 | 138.31 | ··· | ··· | ··· | ··· |
| 2 | 20.63 | 41.85 | 82.68 | 217.84 | 404.10 | ··· | 1936.42 |
| 3 | ··· | 24.92 | 63.01 | 127.11 | 275.48 | 572.01 | ··· |
| 4 | 8.34 | 15.93 | 35.79 | 76.62 | 171.57 | 371.44 | ··· |
| 5 | 4.11 | 11.05 | 22.92 | 48.71 | 105.15 | 242.88 | ··· |
| 6 | 3.23 | 7.64 | 14.40 | 30.90 | 62.78 | 153.05 | 345.98 |
| 7 | 1.99 | 5.27 | 9.37 | 19.56 | 39.92 | 91.38 | 210.97 |
| 8 | 1.21 | 3.58 | 6.09 | 12.60 | 24.89 | 60.05 | 131.81 |
| 9 | 0.73 | 2.30 | 3.95 | 8.14 | 15.51 | 37.95 | 93.25 |
| 10 | 0.47 | 1.38 | 2.62 | 5.31 | 9.83 | 24.18 | 54.80 |
| 11 | 0.29 | 0.66 | 1.71 | 3.38 | 6.47 | 15.47 | 35.05 |
| 12 | 0.18 | 0.30 | 1.02 | 2.10 | 4.10 | 10.07 | 24.28 |
| 13 | 0.12 | 0.18 | 0.65 | 1.31 | 2.58 | 6.32 | 13.39 |
| 14 | 0.08 | 0.11 | 0.42 | 0.85 | 1.65 | 3.84 | 8.59 |
| 15 | 0.05 | 0.07 | 0.26 | 0.53 | 1.04 | 2.39 | 5.39 |
| 16 | 0.03 | 0.04 | 0.16 | 0.34 | 0.66 | 1.53 | 3.43 |

Note: Table 9 presents the average parallax in mili-arcseconds of different spectral types at different magnitudes.

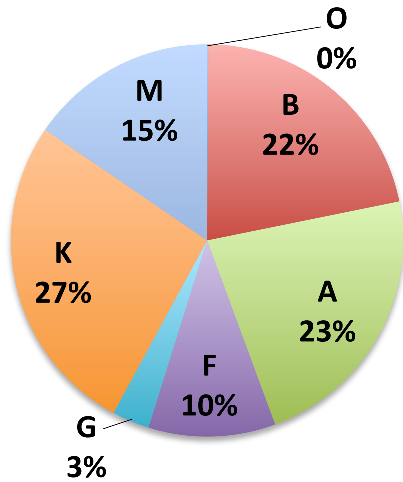


Figure 1: Spectral Distribution for $5^{th}$ Magnitude Stars

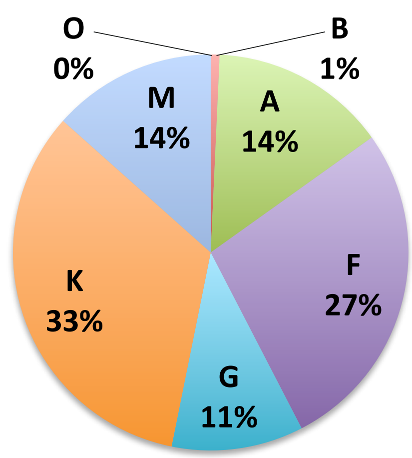


Figure 2: Spectral Distribution for $10^{th}$ Magnitude Stars

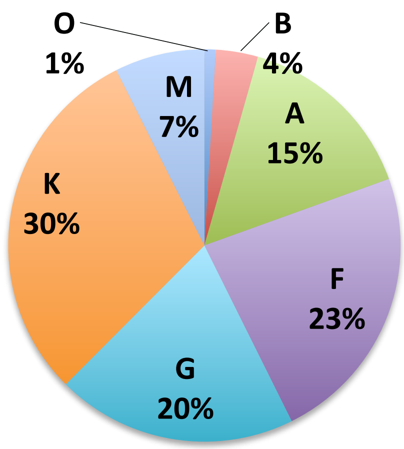


Figure 3: Spectral Distribution for $12^{th}$ Magnitude Stars

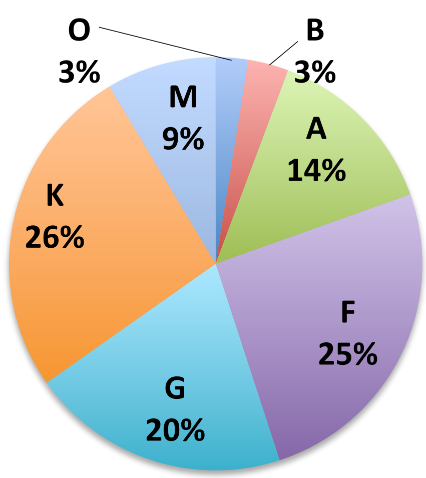


Figure 4: Spectral Distribution for $14^{th}$ Magnitude Stars

## 4. Discussion

Because several smaller catalogs compose NOMAD, each having its own specific range in magnitude, the accuracy of the photometric data is not uniform. Most notably, the Tycho-2 Catalogue (Høg et. al. 2000) provides accurate photometric data through the $11^{th}$ magnitude, after which, the U.S. Naval Observatory CCD Astrograph Catalog (UCAC2) (Zacharias et. al. 2004) has no B or V-band phtotometric data, and the USNO-B (Monet et. al. 2002) catalogs have significantly less-precise photometric information. The drop in accuracy of the B and V-band data after $11^{th}$ magnitude may cause the drop in the number of M stars after the $11^{th}$. M stars have the highest B-V value of the spectral types concerned, a bias toward a smaller B-V value could cause the number of M stars with magnitudes fainter than the $11^{th}$ to decrease. That same bias may also be responsible for the increase in O stars beginning at that same magnitude because O stars have the smallest B-V value of the spectral types concerned.

According to Table 4, however, the proportion of stars with photometric data does not drop significantly during the catalog transition. Thus, the spectral distributions, distances, and photometric parallaxes at magnitudes fainter than V=11.5 are not as accurate as those at brighter magnitudes, which poses a problem because the $12^{th}$ and $14^{th}$ magnitudes are important to the J-MAPS mission.

While the extra constraints placed on M and K stars excluded many giants and likely many main sequence stars as well, they could not exclude all giants because of the overlap in photometric value limits between main sequence K and M stars and the respective giants. As a result, many of the distances and parallaxes included in the averages listed in Table 8 and 9 are inaccurate. These inaccuracies are most visible at the brighter magnitudes, where some of the values conflict with empirical data from Cox (2000). Some of the distances and parallaxes for K and M stars are not included in the data because they differ from previous data and because they are calculated from only a small number of stars.

## 5. Summary

The distributions, in terms of raw counts and percentages, as well as the average distances and parallaxes of each spectral type per magnitude are reported.

## 6. References

Cox, A. 2000, "Allen's Astrophysical Quantities," AIP Press, Chapters 7 and 15

Høg, E. et al. 2000, AsPH, 355, L27-L30 (Tyco2 Catalog)

Mermilliod, J. C. 1981, AsPH, 97, 235

Monet, D. et. al. 2003, AJ, 125, 984

Schmidt-Kaler, T. et al. 1982, AsPH, 6, 2b

Turner, D. 1981, AJ, 86, 222

Zacharias, N., Urban, S. E., Zacharias, M. I., Wycoff, G. L., Hall, D. M., Monet, D. G., & Rafferty, T. J. 2004, AJ, 127, 3043 (UCAC2)

Zacharias, N., Monet, D. G., Levine, S. E., Urban, S. E., Gaume, R., & Wycoff, G. L. 2004, Bulletin of the AAS, 36, 1418 (NOMAD)